UUCMS. No.

B.M.S COLLEGE FOR WOMEN
BENGALURU-560004

III SEMESTER END EXAMINATION – APRIL 2024

M.Sc. MATHEMATICS - TOPOLOGY-I
(CBCS Scheme-F+R)

Course Code MM103T   **QP Code: 11003**
Duration: 3 Hours   **Max. Marks: 70**

Instructions: 1) All questions carry equal marks.
2) Answer any five full questions.

1. (a) Define an infinite set. Let X be an infinite set and let $x_0 \in X$, then prove that $X - \{x_0\}$ is infinite.
   (b) Show that (i) Superset of an infinite set is infinite. (ii) Subset of a finite set is finite. **(7+7)**

2. (a) Prove that any non-empty subset of a countable set is countable. Also prove that the set of all integers is countable.
   (b) Prove that $N \times N$ is denumerable. **(7+7)**

3. (a) Define metric space. Suppose (X, d) is a metric space, let $d_1(x, y) = \frac{d(x,y)}{1+d(x,y)}$
   be defined on $X \times X$. Prove that d_1 is a metric on X.
   (b) Prove that every closed subspace of a complete metric space is complete.

   (8 + 6)

4. (a) State and prove Cantor's intersection theorem.
   (b) Prove that every complete metric space is of the second category.

   (7 + 7)

5. (a) Define topological space. Prove that every metric space is a topological space.
   (b) Define (i) neighbourhood of a point (ii) limit point of a set. Let $A \subseteq (X, \mathcal{T})$
   then prove that $A \cup D(A)$ is closed, where $D(A)$ is derived set of A.

   (7 + 7)

6. (a) Define interior and boundary of a set. Prove that $(A')^0 = (\bar{A})'$.
   (b) Prove that $\bar{A} = A \cup b(A)$. **(8 + 6)**

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7. (a) Let $(X, \mathcal{T})$ be a topological space. Prove the following
   (i) X is neighbourhood of every point.
   (ii) If A is a neighbourhood of x and $A \subseteq B$ then B is also a neighbourhood of x.
   (b) Show that a bijective function $f: X \to Y$ is homeomorphism if and only if $f(A^0) = [f(A)]^0$ for all $A \subseteq X$.

   (7+7)

8. (a) Show that closure of a connected set is connected.
   (b) Prove that union of family of connected sets with non-empty intersection is connected.
   (c) Give an example to show that a connected space is not locally connected.

   (5+5+4)

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